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Letter to the Editor

The Ratio of Nonstationary Tracer Fluxes Into and Out of a Hollow Circular Cylinder

Recent results on the flux ratio under nonstationary conditions (Sten-Knudsen & Ussing, 1981; Bass & Bracken, 1983) have proved and extended the somewhat surprising conjecture (Ussing, 1978) that the standard flux ratio is independent of time from the very first appearance of each of the two unidirectional outfluxes from a plane membrane, no matter how the diffusion coefficient and the drift velocity of the tracer may depend on the distance from a membrane boundary. Since this result holds only for a membrane with a single transport pathway (i.e., with no heterogeneity transverse to the direction of transport), its operational significance is that when a nonconstancy of the flux ratio is found experimentally, the presence of at least two kinds of transport pathways can be inferred and analyzed (Ussing, Eskesen & Lim, 1981).

While flux ratio analyses have been applied primarily to epithelia, recent developments of techniques and results on the permeability of single perfused capillaries (Crone, Frøkjaer-Jensen, Friedman & Christensen, 1978) raise the possibility of resolving the long-standing conundrums of the transport pathways through endothelia (Crone, 1980) by the study of flux ratios under conditions of cylindrical symmetry. The following extension of the aforementioned new results is therefore of interest.

Let the plane membrane dealt with hitherto be folded into a hollow cylinder with cross-sections bounded by two concentric circles, and without change in the initial and boundary conditions pertaining to the flux ratio experiments. If one treats the resulting diffusion-migration problem in cylindrical coordinates by the method used by Bass & Bracken (1983) for the plane membrane, one finds readily that all their results (including those obtained from the convolution theorem) remain valid, provided only that the two unidirectional outflux densities from the infinite plane membrane are replaced throughout by the total unidirectional outfluxes across the inner and outer cylindrical surfaces, respectively. In the flux ratio expressions one thus needs merely to multiply each of the two outflux densities by the radius of the cylindrical boundary that it is crossing. Equivalently, one can retain flux densities but multiply each prescribed boundary concentration by the radius of that cylindrical surface on which it is prescribed.

These results are plausible intuitively: by symmetry all transport is radial, but conservation of tracer within any fixed element of angle introduces the ratio of the two radii as a factor in the flux ratio. The calculation outlined above shows that this remains true for arbitrary radial variations in the diffusion coefficient and in the drift velocity of the tracer.

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